**RANDOM VARIABLES:**

**Expectation of RV:**

Expectation is essentially the weighted average

For two random variables, x and Y, E[X + Y] = E[X] + E[Y]

We want to look at more than just the mean of a RV

We also want to look at the spread of an RV

Standard Deviation = sqrt(variance) captures this well

Tells how far values are from the mean

Variance = average distance from mean

Look at the average of the square of the distance of each value from the mean

= average((distance from average)2)

Need to also look at the weights of the values

Formally: If X is a RV, Var(X) = E[(x-E(x))2], where x is the values the RV can take

If the variance is small, the RV is concentrated around the average

If the variance is large, the RV is spread out

If X and Y are two independent RVs, then Var(X + Y) = Var(X) + Var(Y)

**3 Inequalities:**

Markov’s Inequality:

Suppose we have n students, average is A

How many students scored more than 75?

Suppose X got more than 75

75X <= nA

So, X <= nA/75

Fraction of students who got > 75 = A/75

Formally: Let X be a non-negative RV, then Pr[X>v] <= E[X]/v

Example:

Suppose we toss fair coin n time

Want Pr(we see more than 2n/3 heads) < = \_\_\_\_\_

IE. Pr(X > v)

Let X = number of heads

Pr(X > 2n/3) = E[X]/(2n/3) = n/2 \* 3/2n = ¾

Chebychev’s Inequality:

Pr[abs(X-E[X])>v] <= Var(X)/v2

Example:

Suppose we toss fair coin n times

Want Pr(we see more than 2n/3 heads)

IE. Pr(X > 2n/3) <= ­­­­Pr[X > 2n/3 or X < n/3]

= Pr[abs(x-n/2) > n/6]

= Pr[abs(X – E(X)) > n/6] <= Var(X)/((n/6)2) = 36Var(X)/n

Var(X) = E[(X-E(X))2] = E[(X-n/2)2] = E[X2 + n2/4 – nx] = E[x2] + n2/4 – n2/2

Chernolf Bound:

Suppose you have an opinion poll population

Randomly choose a set of 1000 people, where X support A, and 1000-X support B

A higher number of data estimate points is better

Example:

Suppose X1 … Xm are [0 to 1] random variables that are mutually independent

Suppose E[X] is the same for all X = P

What happens if we add up Xi and divide by m (IE. Sum(Xi)/m)?

Sum(Xi)/m = P

We want to know Pr(|sum(Xi)/m – P| > Pδ)

Chenolf says that this probability is <= 2 \* e ^ (-Pδ2m/2) = 2 ^ (-Pδ2m)

Example:

We toss a fair coin n times

Pr(we see more than 2n/3 heads)

X = # of heads

Pr(X > 2n/3)

Xi = 1 if ith toss is H

E[Xi] = ½

All Xi are mutually independent

Write as Pr[sum(Xi)/n > 2/3] = Pr[sum(Xi)/n – ½ > 1/6)

P\*δ = 1/6 = ½ \* δ, so δ = 1/3

So, the Probability is <= 2\*e ^ (-1/2 \* 1/9 \* n/2) = 2 \* e ^ (-n/36)